CALCULATING THE EFFECT OF INTERACTION BETWEEN TWO PARALLEL WHIRLING JETS

R. B. Akhmedov, D. M. Akhmedov, and T. B. Balagula

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The results of an experimental study of the interaction between two parallel whirling jets are presented. The superposition principle is established for the tangential-velocity field of two parallel vortices.

The development of new methods for regulating furnace processes so that the boiler aggregate will perform as required within a wide range of load levels and during the simultaneous–separate combustion of different fuels has led to the formation of a procedure for controlling the fuel-gas dynamics on the basis of the interaction between twin vortices [1].

The effect of this interaction has been studied on a furnace model 1200 × 1300 × 2700 mm in size with two reversible burners d = 100 and d = 150 mm in diameter. The burner system was designed with provision for the two jets to whirl either in the same or in opposite directions. The amount of twist and the direction of whirling could be varied by changing the position of a cylindrical slide valve in each burner. The design parameter n defining the twist according to the method in [2] was varied within the range 0.73–2.23. This parameter had been defined in terms of the spreader system geometry:

$$n = \frac{d^2}{Lm\varepsilon} \cos\alpha, \tag{1}$$

where d denotes the diameter of the cylindrical channel, L the active length of the spreader (along the burner axis), m the mumber of vanes, ε the shortest distance between vanes, and α the angle between a vane and the tangent to the spreader hub passing through the tip of that vane.

The velocities of the whirling jets were measured with a cylindrical probe in a plane through the burner axes, at distances from the nozzle x/d = 0 to 6. As a result, it was possible to establish the optimal center-to-center distances between the burners for producing an interaction field between the two parallel vortices and to evaluate the effect of the jet twist on this interaction.

The essence of the investigated phenomenon is as follows.

In a system which contains two parallel turbulently whirling furnace jets, already at a definite distance apart depending on their twists and initial momenta, in the region separating them there appears in the plane through their axes a resultant field of tangential velocities equal to the algebraic sum of the respective tangential velocities of the two individual vortices. The character of this resultant tangential-velocity field depends on the sense of rotation of the interacting jets.

When the jets are whirling in opposite directions, then the resultant tangential velocity is equal to twice the tangential velocity of either jet (if both jets are of the same dimensions and intensities) at half the distance R between the two jet axes. When the jets are whirling in the same direction, then the resultant tangential velocity is zero at this distance and has opposite signs on both sides of the point R/2, these signs depending on the direction of whirling.

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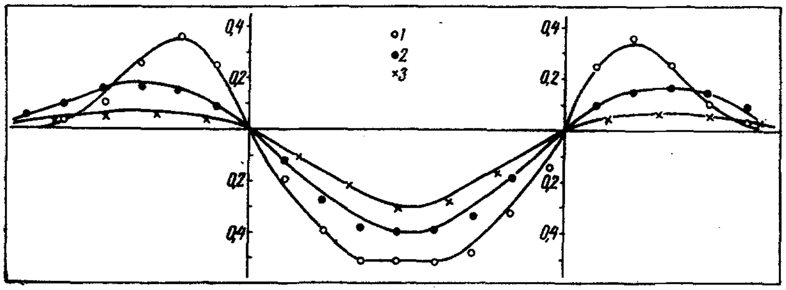

Fig. 1. Experimentally determined distribution of tangential velocities in the region between the two jet axes: 1) x/d = 1.5; 2) 2.5; 3) 3.5.

The results of experimental studies concerning the interaction between twin vortices are shown in Fig. 1. The curves of the distribution of the resultant tangential velocities plotted for distances of a few nozzle diameters confirm these conclusions.

The field of interaction of the vortices may, to the first approximation, be described on the basis of an analysis of two ideal vortices.

As is well known, the tangential-velocity distribution in the field of a vortex (inside regions of radii R_1 and R_2) and outside it is as shown in Fig. 2. The superposition principle applies in the plane passing through the axes of both ideal vortices and, as a corollary, the tangential velocities add algebraically:

$$U_\varphi = \omega_1 \frac{R_1^2}{r} + \omega_2 \frac{R_2^2}{R-r}. \tag{2}$$

The resultant tangential-velocity field obtained by combining ideal vortices in accordance with Eq. (2) cannot quite adequately explain the interaction between real vortices propagating in a viscous medium, since their interaction is accompanied not only by turbulent diffusion in the perpendicular plane but also by convective as well as diffusive displacement in the direction of the jet flow.

Calculating the interaction between two vortices is a complicated mathematical problem a solution to which can, in principle, be arrived at on the basis of boundary-layer equations. It is to be noted, however, that, since boundary-layer equations are nonlinear, they do not directly yield a superposition of velocity fields of interacting vortices.

Of all the known semiempirical methods of solving jet problems, the method most applicable for this particular case is that used for solving the equivalent problem in heat-conduction theory [3]. The gist of this method is that, instead of solving nonlinear boundary-layer equations for the velocity and pressure components, the linear equation of transient heat conduction

$$\frac{\partial B}{\partial \xi} = \frac{\partial^2 B}{\partial \eta^2} + \frac{1}{\eta} \cdot \frac{\partial B}{\partial \eta}, \tag{3}$$

is solved in a fictitious (ξ, η)-space for the variable B which is a definite function of velocity and pressure components.

It has been shown in [3] that, for a large class of self-simulating and certain nonself-simulating flows, mapping from the (ξ, η)-space into the real (x, y)-space is effected as follows:

$$\xi = \xi(x), \qquad \eta = y. \tag{4}$$

The function $\xi(x)$ is determined by comparing the solution to Eq. (3) with the experimental data at a fixed value of y (e.g., at y = 0).

The method of the equivalent heat-conduction problem has been used successfully in calculating the aerodynamics of straight jets with the flow density ρU^2 as the variable B [3].

The calculation of whirling jets is more complicated. Without dwelling now on the axial velocities in a whirling jet [4, 5], we will consider in more detail the tangential velocities. It was originally suggested in [5] that the tangential velocity U_φ be determined from Eq. (3) with $B = \rho U_\varphi^2$. As has been shown subsequently in [4], however, it is not legitimate to use Eq. (3) in calculating ρU_φ^2 because of the incompatibility between Eq. (3) and the boundary condition $\rho U_\varphi^2|_{y=0} = 0$ which the variable ρU_φ^2 must satisfy. It

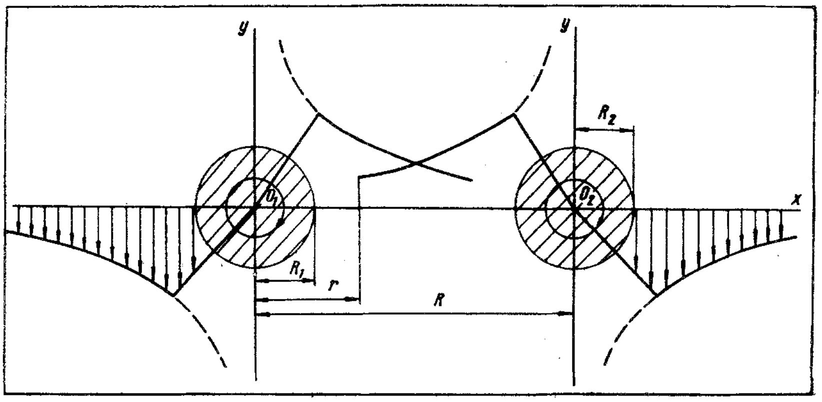

Fig. 2. Distribution of the velocity U_φ in the field of an ideal vortex and outside it.

has been shown in [4] that the diffusion equation (3) is satisfied not by ρU_φ^2 but by the vortex vector component Z_x, which is defined as

$$Z_x = \frac{1}{y} \cdot \frac{\partial (yU_\varphi)}{\partial y}. \tag{5}$$

The tangential velocity U_φ is then determined from the diffusion equation for the vortex vector Z_x. The procedure for calculating U_φ according to [4] is based on the following.

It has been proved rigorously in [6], based on the hydrodynamics of a viscous laminar flow, that vortex $\vec{Z} = \operatorname{curl} \vec{U}$ satisfies the equation of transient heat conduction

$$\frac{\partial Z}{\partial t} = \nu \Delta Z. \tag{6}$$

The vector equation (6) is equivalent to three scalar equations for the vortex components Z_x, Z_y, Z_φ. A conversion from Eq. (6) for the Z_x vortex component to the equivalent heat-conduction problem will, in the axially symmetrical case, yield

$$\frac{\partial Z_x}{\partial \xi_Z} = \frac{\partial^2 Z_x}{\partial y^2} + \frac{1}{y} \cdot \frac{\partial Z_x}{\partial y}. \tag{7}$$

The initial condition for Eq. (7) is given as $Z_0|_{\xi_Z=0} = Z_0(y)$ and the boundary condition is defined as $Z_x \to 0$ at infinity. The solution to Eq. (7) is known [3] and can be written as

$$Z(\xi_Z, y) = \frac{\exp\left(-\frac{y^2}{4\xi_Z}\right)}{2\xi_Z} \int_0^\infty Z_0(r) \exp\left(-\frac{r^2}{4\xi_Z}\right) I_0\left(\frac{ry}{2\xi_Z}\right) r\,dr. \tag{8}$$

Here I_0 is the first-order Bessel function, r is the variable radius in the plane $\xi = 0$. From the relation between Z_x and U_φ according to (5) we find the tangential velocity:

$$U_\varphi = \frac{1}{y} \int_0^y Z_x y\,dy. \tag{9}$$

The solution for U_φ is then

$$U_\varphi = \frac{1}{2\xi_Z y} \int_0^y \exp\left(-\frac{y^2}{4\xi_Z}\right) y \left[\int_0^\infty Z_0(r) \exp\left(-\frac{r^2}{4\xi_Z}\right) \cdot I_0\left(\frac{ry}{2\xi_Z}\right) r\,dr\right] dy. \tag{10}$$

The procedure for calculating U_φ according to Eq. (10) has been shown in [4] and is as follows.

The initial $Z_0(r)$ profile at the nozzle throat is approximated by a piecewise discrete function. Here $Z_0(r)$ is found according to Eq. (5):

$$Z_0(r) = \frac{1}{r} \cdot \frac{\partial (rU_\varphi)}{\partial r} = \frac{U_\varphi}{r} + \frac{\partial U_\varphi}{\partial r}, \tag{11}$$

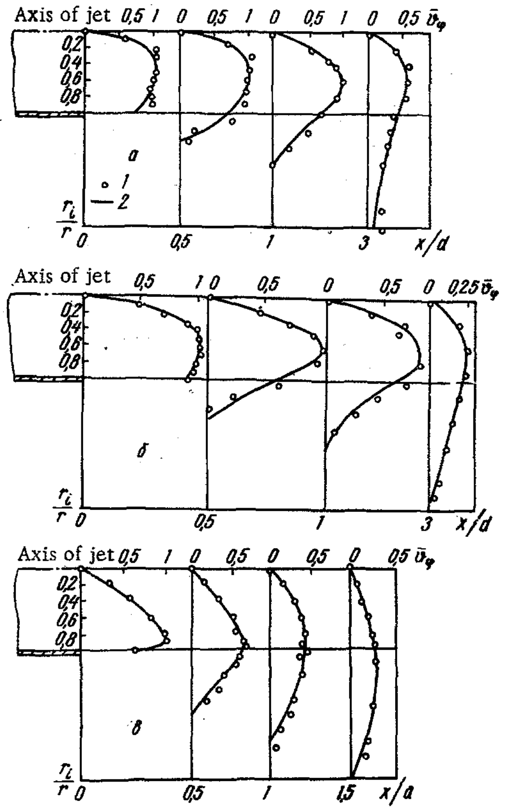

Fig.3

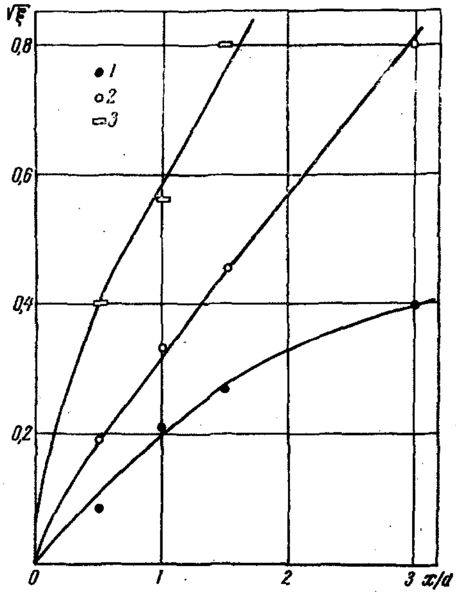

Fig.4

Fig.3. Rotational-velocity field in a single whirling jet: a) n = 0.32; b) 0.51; c) 0.74; 1) test data; 2) theoretical data.

Fig.4. Function $\sqrt{\xi}(x)$ for a twisted jet: n = 0.73 (1), 1.09 (2), and 2.23 (3).

the derivative $\partial U_\varphi/\partial r$ being replaced by the ratio of increments. The expression for Z_x can then be reduced to the sum of so-called P-functions:

$$P(\xi, y) = \frac{1}{2\xi} \exp\left(-\frac{y^2}{4\xi}\right) \int_0^1 \exp\left(-\frac{r^2}{4\xi}\right) I_0\left(\frac{ry}{2\xi}\right) r dr, \tag{12}$$

which have been tabulated in [3]. After Z_x is determined, velocity U_φ will be found by a numerical integration of (9).

Calculations and test results for a single jet with various twists are compared in Fig.3. The calculated results are seen here to agree closely with the test data, despite the presence of a backstream region.

According to the equivalent-problem method, a theoretical calculation of tangential-velocity profiles requires a transformation of coordinates from ξ, η to x, y. Here the $\xi(x)$ relation was found by comparing theoretical and experimental values along the jet axis. The $\sqrt{\xi}(x)$ curve is shown in Fig.4. One can see that, as the twist n is increased, $\xi(x)$ increases too, and this indicates a stronger turbulization of the jet at larger twist values. The magnitude of $\sqrt{\xi}(x)$ increases also with the distance from the nozzle throat.

The suggested procedure for calculating U_φ has been checked out also on a set of two concentric jets with the inner air jet whirling and surrounded by a thin gas jet. The calculated results agreed closely with test data. Thus, this method of determining U in whirling jets has been proved applicable to the entire gamut of twisted jets found in practice.

The use of Eq.(3) for vortex Z_x results in a linear relation between Z_x and U_φ according to Eq.(5) and, consequently, the tangential velocity U_φ as well as Z_x satisfies the linear differential equation. It follows, then, that the superposition principle should apply to U_φ fields produced by several sources, since the equation which describes the vortex magnitude is linear. As is well known, such equations can be solved by the method of sources. This leads to the immediate conclusion that the resulting field due to interaction between two jets can be found by adding the fields produced by each jet alone. Therefore, a calculation of the resultant tangential-velocity field can be reduced to an algebraic addition of the tangential velocities in the individual interacting jets, as determined by the method in [4].

An application of the method of the equivalent heat-conduction problem does not automatically imply that all quantities describing a jet are subject to the superposition principle. In the case of straight jets, for example, not the axial velocities but the flow densities ρU^2 are additive algebraically. By the same token, if Eq. (3) were applied to ρU_φ^2, as suggested in [5], the tangential velocities could not become additive algebraically.

In order to calculate the observed effect of interaction between two parallel vortices, the equation of heat conduction must be applied directly to the quantity U_φ or to any other quantity linearly related to it. This proves that the method proposed in [4] for calculating U_φ on the basis of vortex diffusion is sufficiently accurate for the description of the tangential-velocity fields in complex jet streams.

NOTATION

U, U_φ	are velocity components (axial and tangential), m/sec;
ω	is the angular velocity, sec^{-1};
R	is the center-to-center distance between vortices, m;
B	is the transferred substance;
ρ	is the density, kg/cm^3;
ν	is the kinematic viscosity;
t	is the time;
∇	is the Laplace operator;
ξ, η	are fictitious coordinates (longitudinal and transverse);
n	is the design parameter characterizing the twist, equal to the ratio of the momentum at the burner entrance to the momentum at the burner throat.

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